Application of Simultaneous Stochastic Optimization at Large Copper Mining Complex with Geological and Equipment Uncertainty

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Abstract

The simultaneous stochastic optimization of mining complexes aims to optimize the different components in a single optimization model under grade, material type and, in the present work, equipment production uncertainty, capitalizing on the synergies between the various components and the quantified variability and uncertainty of the materials mined, to meet production targets and capacities better. The uncertainty and variability associated with the different material sources are incorporated in the optimization model using stochastic simulations, also employed to quantify the uncertainty related to equipment production, generated conditional to historical mining and processing production data collected from different equipment in the mining complex. The current work presents an application to integrate uncertainty and decisions about mining capacities dictated by the available mining equipment (e.g. trucks and shovels) and crushers' production capacities in the simultaneous stochastic optimization model. The application of the approach at a large copper mining complex composed of two deposits, three stockpiles, five crushers, three mills, two leach pads, a waste dump and several different mining equipment types and models indicates that the stochastic schedule has higher chances of meeting the production targets and capacities while achieving a substantial increase in the net present value (NPV) of the mining complex when compared to the conventional plan.

Keywords: *Mining complex; Stochastic simulations; Simultaneous stochastic optimization.*

1 Introduction

Mining operations are most commonly comprised of multiple components such as mineral deposits, stockpiles, mills, leach pads, waste dumps, several different types of equipment and customers. These elements together represent a mineral value chain, also known in the literature as a mining complex [11; 18; 22]. The mining complex can be viewed as a transfer function that involves complex interactions, where the material extracted from the mines flows through different components (e.g. from mines to crushers and later to mills) and is transformed into sellable products (e.g. metal concentrate) near the end of the chain. Since this flow involves complex non-linear transformations, it becomes difficult the use of conventional optimizers to tackle the problem and simultaneously optimize the entire complex, with the traditional optimizer's approach being characterized by solving each component individually or sequentially (e.g. optimizing the extraction sequence separately from the quality of material at mills). Such an approach does not benefit from the interaction and cooperation of different components, leading to suboptimal solutions for the value chain [18].

Nevertheless, past efforts directed to the simultaneous optimization of mining complexes aimed to include more decisions in the optimization process, such as the model presented by Hoerger et al. [13] for Newmont's Nevada mining complex that simultaneously optimizes the timing of open-pit layback, underground stope development, capital expenditure, the timing of processing plant start-ups and shutdowns, and, material routing decisions. Whittle and Whittle [33] presents a global optimization model that includes optimization of extraction sequence, mining rate, cut-off grade policy, processing path selection and stockpiling strategy, which is further developed in Whittle [32], expressing grades as an additive property and allowing the integration of other aspects of stockpiling and transportation in the global asset optimization model, generating the Prober C algorithm. However, besides the typical limitations of the conventional approaches, such as aggregation of mining blocks and lack of genuinely simultaneous optimization of different components, the main drawback of the models mentioned above is their inability to account for the uncertainty from various sources, especially uncertainty in grades and material supply since conventional optimization techniques rely on the use of a single estimated orebody model as input to the optimization process, which is unable to reproduce the in-situ variability of the deposit's grades and is but a smooth representation of the orebody [4]. Material supply uncertainty has long been recognized as the primary cause of technical risk in mining operations [31], leading to unexpected deviations in production targets [5; 7; 8; 25].

The simultaneous stochastic optimization of mining complexes overcomes the limitations of the previous models by considering one single optimization model to simultaneously optimize all different components of a mining complex under uncertainty. Montiel and Dimitrakopoulos [18] propose a model that simultaneously optimizes block extraction, destination, processing streams utilization, operating modes and transportation alternatives under grade and material type uncertainty, later extended in Montiel and Dimitrakopoulos [19, 20]; Montiel et al. [21] to include underground mines and a metaheuristic algorithm to solve the large optimization model of mining complexes.

Goodfellow and Dimitrakopoulos [11] and [10] present a model that simultaneously optimizes block extraction decisions, destination policies based on block multi-element clustering and processing stream utilization strategies under grade and material type uncertainty, later extended in Goodfellow and Dimitrakopoulos [9] to include investment decisions. These simultaneous stochastic optimization models can truly capitalize on the existing synergies between the mining complex components while focusing on the value of the products sold rather than the value of mining blocks and being capable of integrating the uncertainty in material supply and the grade variability, producing mining schedules better suited to meet blending targets and production forecasts, which leads to higher project net present values (NPV). Spatial grade variability and uncertainty are quantified by stochastically generated orebody simulations [12; 14; 17; 28].

Recent work has further extended and developed the simultaneous stochastic optimization of mining complexes framework [3], quantifying uncertainty from different sources [15; 29]. Efforts to include equipment production uncertainty in a stochastic optimization model were more recently proposed by Quigley and Dimitrakopoulos [23] and Both and Dimitrakopoulos [1], works focused on a stochastic optimization of short-term production scheduling with equipment allocation. In the proposed models, the uncertainty in equipment production is also quantified by equiprobable stochastic simulations, with Quigley and Dimitrakopoulos [23] generating the simulations conditional to equipment historical data by Monte Carlo methods. Both and Dimitrakopoulos [1] show the benefits of simultaneously optimizing the mining complex with equipment decisions, which generates savings in costs related to equipment movements and better equipment utilization. However, short-term scheduling is constructed in lower timescales. It aims to develop an operational plan for sub-periods of the initial long-term production scheduling, for which the extraction of given areas (or blocks in a given period of the long-term plan) are assumed to be completely mined. Since later planning and optimization relies on these assumptions, having a long-term production schedule that accounts for equipment uncertainty and respects the equipment production capacities can prove beneficial, given that both initial forecasts and later short-term optimization would be based on schedules that already account for realizable equipment production.

The present work aims to apply the simultaneous stochastic optimization framework to incorporate the equipment production uncertainty and, along with geological uncertainty, generate long-term schedules better suited to meet production targets and capacities. In the following sections, the method utilized is first presented. An overview of the copper mining complex used in the study is shown, with a brief review of the risk analysis of the mining complex current conventional plan performed under joint equipment and geological uncertainty done in Resende Silva and Dimitrakopoulos [27]. Results and discussion are presented next, and conclusions follow in the last section.

2 Stochastic scheduling optimization method

The approach proposed herein is based on the model for simultaneous optimization of mining complexes under uncertainty proposed in Goodfellow and Dimitrakopoulos [11], which is extended in the present work to include uncertainty and constraints related to equipment production, i.e. mining and crusher capacities in the present case. The work presented highlights the applied aspects of the method with an application at a large copper mining complex with over 4 million binary variables. First, definitions, notation and the different decision variables are provided and discussed. Next, the objective function and constraints used in the present work to include uncertainty related to equipment production in the context of simultaneous optimization of mining complexes are outlined.

2.1 Definitions, notation, and decision variables

The notations used in this section are as follows: $\mathbb{M}$ represents a group of mines, where $\mathbb{B}_m$ represents the set of mining blocks b for a given mine m and $MC_{b,t}$ represents the mining cost of block b in period t. To enable access to block b, the extraction of its over-lying blocks, represented by a set $\mathbb{O}_b$, must occur before or at period t. The set of scenarios that quantify the joint uncertainty in grades and material types is defined by $\mathbb{S}$. The total number of scheduling periods is represented by $\mathbb{T}$. Extracted material from mines can either be stockpiled, processed after crushing or sent to waste, where $v_{a,i,t,s}$ represents the amount of property a at location i in period t and scenario s. $\mathcal{S}^{SP}$ represents the set of stockpiles, and $SC_{i,a,t}$ represents the stockpiling cost of stockpile $i \in \mathcal{S}^{SP}$ for property a in period $t \in \mathbb{T}$. $RH_{i,a,t}$ denotes the cost of re-handling material from a given stockpile $i \in \mathcal{S}^{SP}$ for property a in period $t \in \mathbb{T}$. The set of crushers is denoted by $\mathcal{S}^C$ with $CR_{i,a,t}$ representing the crushing cost of material in crusher $i \in \mathcal{S}^C$ for property a in period $t \in \mathbb{T}$. $\mathcal{P}$ represents the set of processing destinations (e.g. mills and leach pads), $PF_{i,a,t}$ represents the profit for recovered product a within processing destination $i \in \mathcal{P}$ in period $t \in \mathbb{T}$ and $PC_{i,a,t}$ represents the processing cost for processing destination $i \in \mathcal{P}$ for property a in period $t \in \mathbb{T}$. Properties such as metal tonnage, rock tonnage and ore tonnage are represented by $\mathbb{H}_p$ and are calculated by adding amounts of metal, rock and ore tonnages processed at different locations in the mining complex, being called primary attributes. Properties such as copper head grade and recovered metal are calculated based on the primary attributes reaching the different locations, represented by $\mathbb{H}_s$ and called hereditary attributes, i.e. the mass of different products recovered. To quantify the uncertainty for both the materials and attributes, it is assumed that each block $b \in \mathbb{B}_m$ has a simulated material classification or attributes defined by $\beta_{a,b,s}$ $\forall a \in \mathbb{H}_p$, $\forall s \in \mathbb{S}$.

Moreover, a mining complex usually operates following production targets for different attributes, such as material quality and capacity targets. $\mathbb{P}_g$ represents attributes subject to the material quality targets, the set $\mathbb{P}_c$ represents attributes subject to capacity targets, and $\mathbb{P}_e$ represents attributes subject to the targets concerning the mining and crusher capacities, the latter concerning the equipment whose uncertainty is being added. Given these targets, $c^+_{a,i,t}$ and $c^-_{a,i,t}$ represent the penalty costs associated with deviations from maximum/upper and minimum/lower production targets, respectively, for property a in each period $t \in \mathbb{T}$. Profits and costs from operations are discounted using an economic discount factor r_{ec} (e.g., $PF_{i,a,t} = PF_{i,a,1}/(1 + r_{ec})^t$), while the cost of deviation from targets is also discounted using a risk discount rate r_{risk} (e.g., $c^+_{a,i,t} = c^+_{a,i,1}/(1 + r_{risk})^t$) defined as in Dimitrakopoulos and Ramazan [6] and Ramazan and Dimitrakopoulos [24], and whose objective is to defer the risk of not meeting production targets to later years when more information is available.

Mineability targets are also included in the model to ensure that the production schedules are practically mineable, where W_b and V_b are the specified mining width and sink rate, respectively. Then $d^{smooth}_{b,t}$ is defined as the number of blocks scheduled in different periods than b inside a given mining width around b, with $c^{smooth}_{b,t,}$ being the penalty cost associated with not scheduling blocks inside the mining width in the same mining period [6; 29]. Similarly, $d^{sink}_{b,t,v}$ is defined as the number of blocks scheduled in the same period as b inside a given vertical window, with $c^{sink}_{b,t}$ being the penalty cost associated with scheduling more blocks than allowed inside the mining sink rate [2; 29].

The model proposed by Goodfellow and Dimitrakopoulos [11] presents three different types of decision variables: (i) mine extraction sequence variables ($x_{b,t} \in \{0, 1\}$), which define whether

(1) or not (0) a block $b \in \mathbb{B}_m$ is extracted in period $t \in \mathbb{T}$; (ii) cluster destination policy variables ($z_{c,j,t} \in \{0, 1\}$), which define whether (1) or not (0) cluster $c \in \mathcal{C}$ is sent to one of the possible destinations j in period t for a given material type, where the cluster destination policy is based on clusters defined over multiple elements of interest, and; (iii) processing stream utilization variables ($y_{i,j,t,s} \in [0, 1]$) defining the proportion of material sent from a location i to a subsequent location j in period t and under scenario s. Beyond the decision variables, other continuous variables that keep track of deviations from different targets are also included in the model, with surplus variables $d^{+}_{a,i,t,s}$ representing the excess over maximum targets $U_{a,i,t}$ for property a in period t and scenario s. Conversely, shortage variables $d^{-}_{a,i,t,s}$ represent shortage from minimum targets $L_{a,i,t}$ for property a in period t and scenario s. For the simulated mining and crusher capacities, these values are represented herein by $CAP^{z}_{a,i,t}$ for property $a \in \mathbb{H}_p$ in period $t \in \mathbb{T}$ and a given probability zone $z \in \mathcal{Z}$ within a location $i \in \mathbb{M} \cup \mathcal{S}^c$.

2.2 Objective function and constraints

The objective function (equation (2.1)) of the stochastic model is a two-stage function that maximizes the value of the products generated from a mining complex and delivered to customers or spot market, while minimizing the deviations from capacities, blending and mineability targets, under grade, material type and production uncertainty. Part I in the objective function represents the profits of different products produced and sold. Part II is the processing cost of the material at the various processing destinations. Part III describes the crushing cost of crushers. Part IV relates to the stockpiling costs, and Part V represents the cost of re-handling material from different stockpiles. Part VI relates to the cost of deviations from the material quality targets at the various processing destinations (e.g. target copper grade). At the same time, Part VII represents the deviations from target capacities at the different processing destinations and stockpiles. In the same way, Part VIII represents the deviations from simulated mining and crusher capacities within a given probability zone $z \in \mathcal{Z}$. Part IX relates to the mining cost at the different mines, and Parts X and XI represent the costs associated with the mineability targets aiming to smooth the schedules.

$$
\begin{aligned}
\max \Bigg\{ \frac{1}{|\mathbb{S}|} \Bigg\{ \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} \Bigg\{ & \sum_{i\in\mathcal{P}} \sum_{a\in\mathbb{H}_s} \underbrace{PF_{a,i,t} \cdot v_{a,i,t,s}}_{Part\ I} - \sum_{i\in\mathcal{P}} \sum_{a\in\mathbb{H}_p} \underbrace{PC_{a,i,t} \cdot v_{a,i,t,s}}_{Part\ II} \\
& - \sum_{i\in\mathcal{S}^C} \sum_{a\in\mathbb{H}_p} \underbrace{CR_{a,i,t} \cdot v_{a,i,t,s}}_{Part\ III} - \sum_{i\in\mathcal{S}^{SP}} \sum_{a\in\mathbb{H}_p} \underbrace{SC_{a,i,t} \cdot v_{a,i,t,s}}_{Part\ IV} \\
& - \sum_{i\in\mathcal{S}^{SP}} \sum_{a\in\mathbb{H}_p} \underbrace{RH_{a,i,t} \cdot v_{a,i,t,s}}_{Part\ V} - \sum_{i\in\mathcal{P}} \sum_{a\in\mathbb{P}_g} \underbrace{c^{+}_{a,i,t} \cdot d^{+}_{a,i,t,s}}_{Part\ VI} \\
& - \sum_{i\in\mathcal{S}^{SP}\cup\mathcal{P}} \sum_{a\in\mathbb{P}_c} \underbrace{c^{+}_{a,i,t} \cdot d^{+}_{a,i,t,s}}_{Part\ VII} - \sum_{i\in\mathbb{M}\cup\mathcal{S}^C} \sum_{a\in\mathbb{P}_e} \sum_{z\in\mathcal{Z}} \underbrace{c^{+}_{a,i,t,z} \cdot d^{+}_{a,i,t,s,z}}_{Part\ VIII} \Bigg\} \Bigg\} \\
& - \sum_{t\in\mathbb{T}} \sum_{m\in\mathbb{M}} \sum_{b\in\mathbb{B}_m} \Bigg[\underbrace{\Big(MC_{b,t} \cdot x_{b,t} \Big)}_{Part\ IX} + \underbrace{\Big(c^{smooth}_{b,t} \cdot d^{smooth}_{b,t} \Big)}_{Part\ X} \Bigg] \\
& - \sum_{t\in\mathbb{T}} \sum_{m\in\mathbb{M}} \sum_{b\in\mathbb{B}_m} \sum_{v\in V_b} \underbrace{\Big(c^{sink}_{b,t} \cdot d^{sink}_{b,t,v} \Big)}_{Part\ XI} \Bigg\}
\end{aligned}
\tag{2.1}
$$

In the current application, the different mines and crushers in the mining complex have their mining and production capacities, respectively, determined by simulations constructed from historical data from shovels, trucks, and crushers production for two years of mining activities. Uncertainty related to equipment production results in inconsistent schedule forecasts with fixed capacities at the mines and crushers. These forecasts may fall short, as shown by Resende Silva and Dimitrakopoulos [27] and consequently lead to fewer metal-re recovered tonnages. A different approach is used in the present work to overcome those issues by integrating the equipment production scenarios into the optimization process and producing schedules more fit to handle this uncertainty.

The mining and crusher capacity simulations are added to the model employing what is herein called *probability zones*. These probability zones are constructed using the different percentiles calculated using the set of simulations for the equipment, where a given probability zone is defined by the area between any two consecutive percentiles and describes the confidence in the realization of a given capacity value, e.g. the 10th percentile from the simulations represents a 90% chance of having that or lower value as reality. Therefore, the 90% zone is the area given by the 10th percentile and the next calculated percentile. For the mined tonnages at the different mines, the single extraction sequence is linked to the simulated capacities using the constraint shown in equation (2.2), where the deviations from the simulated capacity within a given probability zone are calculated and penalized in the objective function, producing a sequence that is capable of obeying the different scenarios. Different penalties are applied to different probability zones $z \in \mathcal{Z}$. The size of the penalty will be translated as the willingness of risk acceptance by the planner or decision-maker, i.e. risk-averse decisions would increase penalties on deviations even within zones of a high probability of capacity realization (e.g., deviations within the 90% zone). In comparison, risk-tolerant decisions would choose to penalize only deviations falling into zones of a low probability of realization (e.g., deviations happening in the 50% or 30% zones). The optimizer then perceives these penalties for deviations from the different probability zones and constructs a schedule that aims to respect the simulated capacities given the risk willingness of the decision-maker.

$$\left(\sum_{b \in \mathbb{B}_m} \beta_{a,b,s} \cdot x_{b,t}\right) - d^{+}_{a,m,t,s,z} - CAP^{z}_{a,m,t} \leq 0, \forall a \in \mathbb{H}_p, m \in \mathbb{M}, t \in \mathbb{T}, s \in \mathbb{S}, z \in \mathcal{Z} \quad (2.2)$$

Moreover, the penalty for deviations within a given zone is applied solely to the deviations inside that zone, meaning that the approach proposed does not overly penalize deviations, which can constrain the problem too much. Similarly, the simulated crusher production capacities are related to the crushed tonnages, as shown in equation (2.3). Other constraints implemented in the model are reserve, slope, capacity, destination policy, processing stream flow, blending and mineability constraints, all detailed in Goodfellow and Dimitrakopoulos [11]; Kumar and Dimitrakopoulos [15].

$$v_{a,i,t,s} - d^{+}_{a,i,t,s,z} - CAP^{z}_{a,i,t} \leq 0, \ \forall a \in \mathbb{H}_p, i \in \mathcal{S}^c, t \in \mathbb{T}, s \in \mathbb{S}, z \in \mathcal{Z} \quad (2.3)$$

The simultaneous stochastic optimization model of mining complexes previously outlined is constructed as a sizeable combinatorial optimization model with millions of binary decision variables and thousands of continuous decision variables. Given its size and the limitations of exact methods in solving such a large optimization problem, metaheuristic algorithms are employed in the solution approach used to solve the model, specifically multi-neighbourhood simulated annealing with an adaptive neighbourhood search algorithm explained in Goodfellow and Dimitrakopoulos [10, 11].

3 Application at a large copper mining complex

This section applies the proposed approach to a large copper mining complex. First, the mining complex is outlined, where general information about its mineral value chain and conventional schedule is given. A constant factor scales economic parameters (e.g. prices and costs), material quality and production targets used in the study for confidentiality reasons. Generic names are also used.

3.1 Overview of the copper mining complex

3.1.1 Mining complex and material flow

The copper mining complex consists of two mines (Mine A and Mine B), having 194,000 and 65,350 blocks, respectively, with a selective mining unit (SMU) size of 25×25×15 m3. The stratigraphic sequence of material is waste rock, followed by copper oxide, mixed and copper sulphide. The mining complex produces copper concentrate and copper cathode as primary products and gold, silver and molybdenum concentrate as secondary products. The material extracted from both mines is classified into 4 different main material types (waste, oxide, high-grade sulphide, and low-grade sulphide) based on geological and grade properties, i.e. concentrations of copper soluble (CuS) and copper total (CuT), and copper soluble to copper total ratio. The extracted material can be sent directly to one out of 10 destinations (5 crushers, 3 stockpiles, 1 bio leach pad for low-grade sulphide and 1 waste dump). Material sent to one of the five different crushers is further directed to one of the three processing mills (high-grade sulphide material) and an acid leach pad (oxide material) that supplies material to the port and a copper cathode plant, respectively, following the flow of material shown in figure 1.

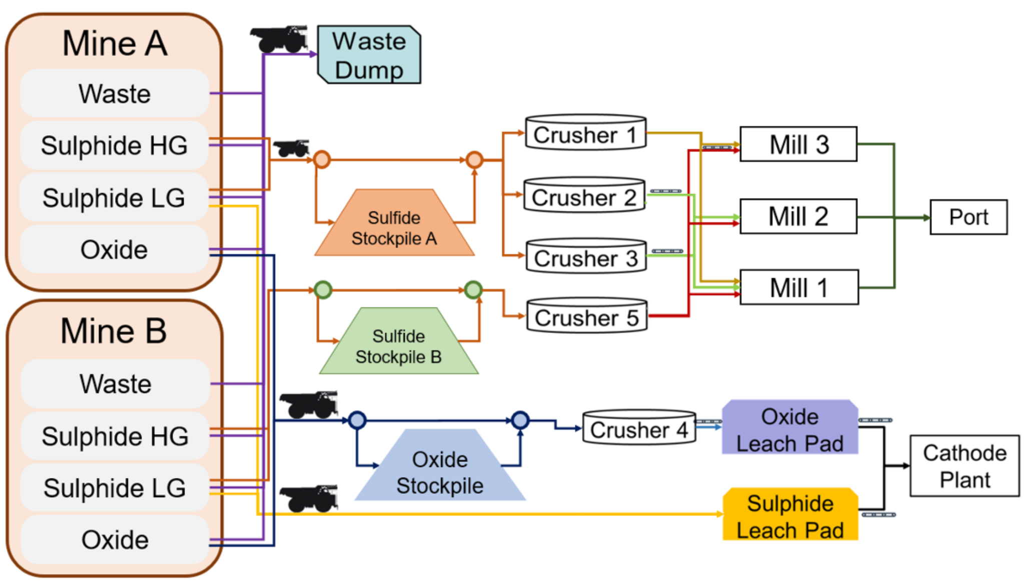


Figure 1: The flow of material at the copper mining complex.

From the orebody simulations provided by the mining complex, 15 stochastic simulations were randomly selected for each mine, quantifying the uncertainty in grades and material types. Grade uncertainty is quantified for two properties: copper soluble (CuS) and copper total (CuT). Figure 2 shows the cross-sections of two randomly selected simulations for the two deposits, where CuT grades are displayed. Compared to the smoothed estimated deposits, the variability in the attribute is easily seen in the simulations.

Besides the 15 orebody simulations, 50 simulations were selected for each mining and crushing capacity from the simulations generated using the equipment historical data and the approach proposed in Resende Silva and Dimitrakopoulos [27], where the cumulative distribution functions (CDFs) for the different information is directly constructed from the historical data and the simulated productions for the various equipment types generated by direct sampling of these CDFs. These 50 equipment simulations were then used to calculate the percentiles employed in developing the different probability zones as previously described. For the current work, the probability zones created were the 90% zone between the 10th and 30th percentiles, the 70% zone between the 30th and 50th percentiles, the 50% zone between the 50th and 70th percentiles, the 30% zone between the 70th and 90th percentiles and the 10% zone above the 90th percentile. Figure 3 shows examples of the mining capacity and crusher production capacity simulations for Mine A and Crusher 5, respectively.

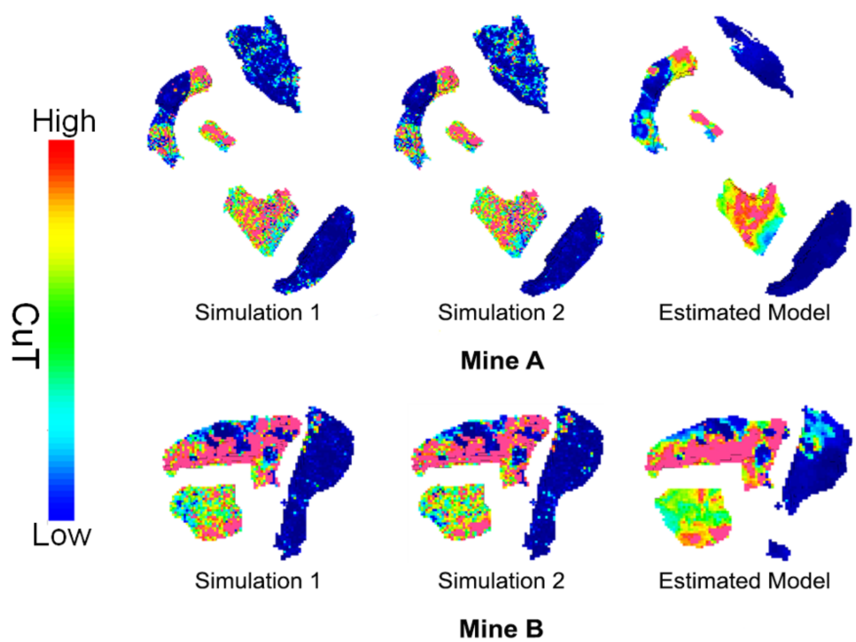


Figure 2: Examples of simulations of the copper total (CuT) grades for the two deposits compared to the estimated model.

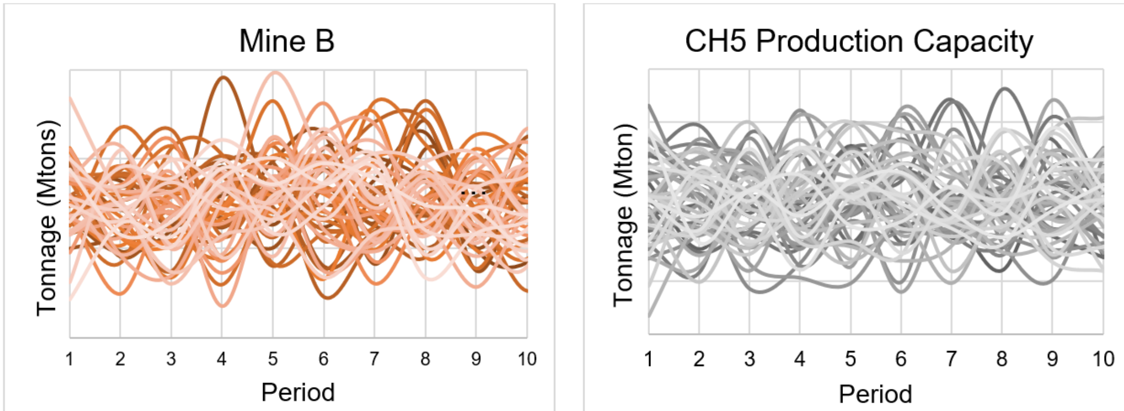


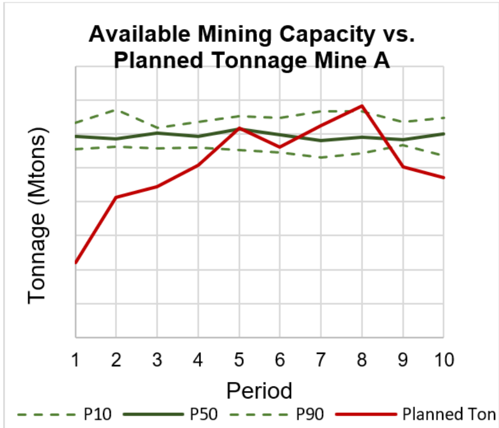
Figure 3: Examples of mining capacity simulations for Mine B (left) and crushing production capacity simulations for Crusher 5 (right) were used to calculate the percentiles and, later, the probability zones.

3.1.2 Risk analysis of conventional long-term production schedule

The long-term mine production plan currently used at the mining complex is optimized using a two-step optimization approach, where: (i) the extraction sequence of multiple mines is optimized independently of each other using Whittle version 4.5.4 [30], a widely used software for strategic mine planning, and; (ii) the destination of the extracted material follows the cut-off grade policy presently used at the mining complex, based on Lane's approach [16; 26], with the utilization of different processing streams defined using a separate optimization model. Also, this two-step optimization process is performed using estimated mineral deposits, shown in figure 2, as is the standard practice in the mining industry. This long-term mining plan of the copper mining complex generated with this two-step approach results in the conventional mine production plan [15].

Since the planned schedule is based on linear programming optimization, it results in a plan with a partial block extraction sequence instead of a mixed-integer programming optimization. In the risk analysis shown herein, the extraction sequence for the risk profiles was first made integer using the same approach as in Resende Silva and Dimitrakopoulos [27]. The results presented next quantify the risk of the schedule in meeting its production forecasts in the presence of uncertainty of the material supply and the capacities of the equipment used and, for comparison reasons, given the conventional plan does not forecast any material sent to the stockpiles in the first 10 years, the same was maintained in the risk analysis. Economic parameters are the same, as shown in table 1.

The output values for a given project indicator (e.g. total tonnage extract, metal produced, cash



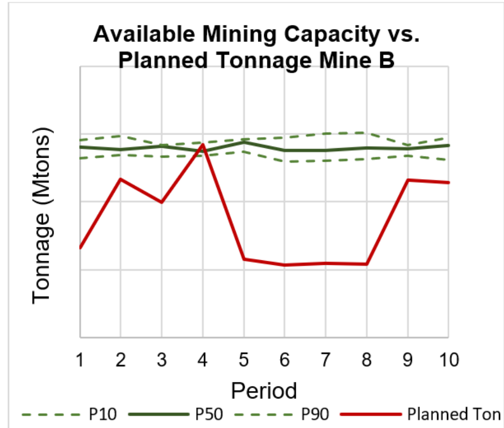


Figure 4: Risk profiles for the simulated total available mining capacities for Mine A (left) and Mine B (right) versus the planned total mined tonnage (red line).

flows) are reported using the 10th, 50th, and 90th percentiles risk profiles (P10, P50, and P90, respectively). P10 represents a 90% chance of having at least that value, which means that the values for 90% of the scenarios are higher than the P10. P50 represents the value at which 50% of scenarios fall above and 50% fall below, and P90 represents a 90% chance of having a value below it.

First, figure 4 shows the simulated mining capacities for the two mines for each of the 10 years LOM. The planned mined tonnage is shown to be below the simulated capacities for most of the periods. However, for periods 7 and 8, the conventional plan has an extraction sequence for Mine A that surpasses the P50 simulated capacity, with the total mined tonnage extrapolating the P90 line in period 8, which indicates that such plan has less than a 10% chance of being able to meet the planned extracted tonnage, impacting the amount of material feeding the different processing destinations, which can be seen in figure 5, where the cumulative difference in total recovered metal tonnages at the three mills and two leach pads can be of up to 7% less than planned at the end of the 10 years (comparing with the P50) and 8% in the first 5 years. This difference is also affected by the uncertainty in the material quality (e.g. copper grades).

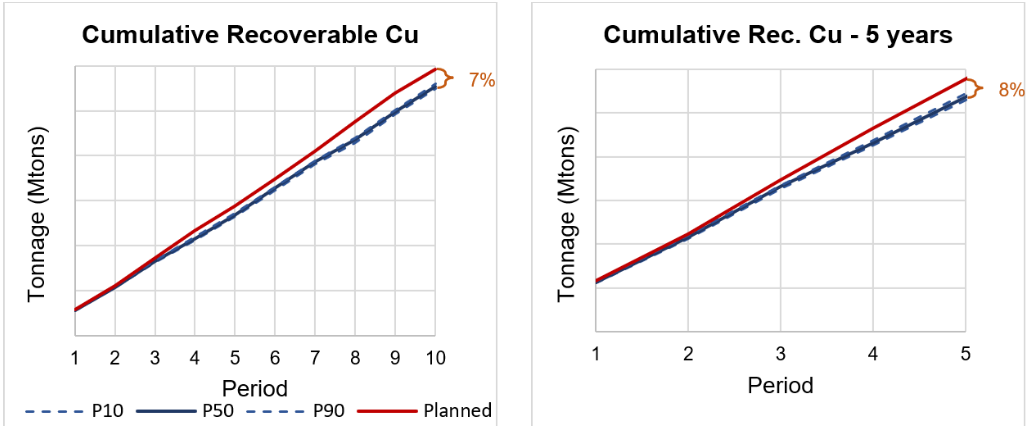


Figure 5: Total cumulative recoverable metal at processing destinations for the LOM (left) and the first 5 years of operations (right).

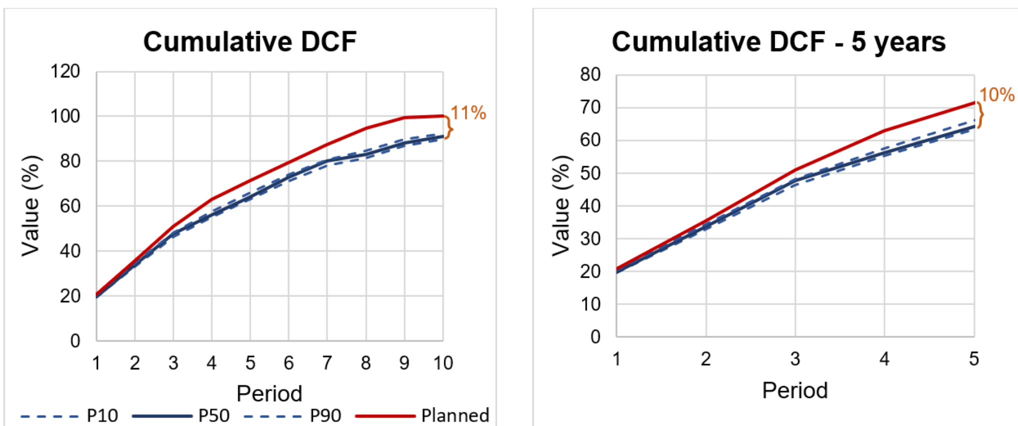


Figure 6: Risk profiles for the cumulative discounted cash flows for the LOM (left) and the first 5 years (right).

The differences between the recoverable copper seen in the initially planned forecasts and the ones shown in the risk profiles come from the inability of the estimated models to reproduce the actual variability of the mineral deposits and the overestimation of ore tonnages mined and processed. The decrease in recoverable copper observed in the risk analysis was caused by the joint uncertainty in material supply. Production capacities have a direct impact on the discounted cash flows (DCF) forecasted, such that at the end of the LOM, the cumulative DCF is seen to be 11% lower than the one predicted by the conventional plan, as shown in figure 6, with the 5 first years already shown to be 10% lower than the initial forecasts. Those differences carry an enormous impact, which can be translated, for a large mining complex, into tens of millions less revenue than expected.

The inability to model the variability of the deposit's properties also results in underestimating the total tonnages of metal sent to the waste, shown in figure 7, where close to 34% more copper is sent to the waste during the 10 years LOM. This difference increases to 41% in the first 5 years.

For a complete comparison, the results shown in the following section present the planned forecasts, called "Planned," and the P50 for the above risk analysis, referred to as "P50-Plan".

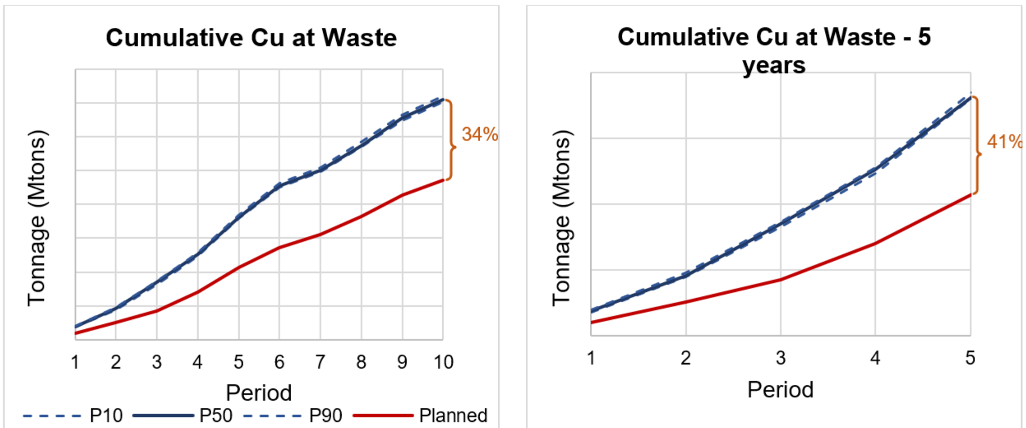


Figure 7: Risk profiles for the cumulative discounted cash flows for the LOM (left) and the first 5 years (right).

3.2 Simultaneous stochastic optimization

Given the simultaneous stochastic optimization of mining complexes described in section 2, the orebody and equipment simulations, the economic and operational parameters shown in table 1 and production targets decided by the mining complex based on studies about the configuration and operating modes of their different processing destinations, the model for the mining complex was optimized. The results of the stochastic optimization are presented next. As previously explained, results are reported in P10, P50, and P90 risk profile percentiles.

The probability zones were defined as previously discussed and shown in figure 8, i.e. the 90% zone is considered as the area between the 10th and 30th percentiles, the 70% zone is considered as the area between the 30th and 50th percentiles, the 50% zone is considered as the area between the 50th and 70th percentiles, the 30% zone is considered as the area between the 70th and 90th percentiles and the 10% zone is considered as the area above the 90th percentile. In the current application, penalties are applied only to deviations above the 50th percentile, i.e. deviations calculated within the 50%, 30% and 10% zones. For crusher capacities, penalties were \$2.5/ton, \$10/ton, and \$30/ton for each unit deviation within the 50%, 30% and 10% zones, respectively. For mining capacities, penalties were \$2.5/ton, \$2.5/ton, and \$10/ton for each unit deviation within the 50%, 30% and 10% zones, respectively.

Attribute	Value
Economic discount rate	8%
Risk discount rate	10%
Copper selling price (US\$/ton metal recovered)	5511.55
Copper selling cost at mills (US\$/ton metal recovered)	571.57
Copper selling cost at leach pads (US\$/ton metal recovered)	551.55
Mining cost (US\$/ton rock) – excluding hauling cost	0.76
Hauling cost based on depth (US\$/ton rock)	0.40 – 1.27
Crushing cost (US\$/ton ore)	0.61
Milling cost (US\$/ton ore)	5.79
Cost re-handling material from stockpile (US\$/ton ore)	0.20
Processing cost Oxide Leach Pad (US\$/ton ore)	6.03
Processing cost Sulphide Leach Pad (US\$/ton ore)	1.14
Recovery copper at mills (Mill 1, Mill 2 and Mill 3)	0.826, 0.830 and 0.804
Recovery copper at leach pads (Sulphide and Oxide Leach)	0.27 and 0.65
Slope angles for Mine A and Mine B	37º and 45º
Mining width (m)	200
Number of clusters for the different material types	30

Table 1: Economic and operational parameters used for the mining complex optimization.

The current conventional long-term schedule was optimized for a LOM of over 100 years. However, the present application focused on the first 10 years of the LOM to have a tractable problem. To produce comparable results, given the conventional plan has intensive waste removal within the 10 years as planned stripping, the pit limits for the first 10 years were flagged, and the stochastic optimizer was set to mine the entirety of blocks within the flagged pit limits. Therefore, the same tonnages were to be extracted, and any improvement in the results is attributed to the method applied.

The first results in figure 9 refer to the scheduled total tons of material to be mined following the extraction sequence for the stochastic plan. Unlike the conventional schedule, the stochastic strategy uses the available mining rates, producing a more even mining production in the first 5 years. As shown below, this does not surpass the P50 mining capacities, as was expected. Such a schedule will, throughout the LOM, have higher chances to be realized once it does not exceed P50 capacity values and plans tonnages in the defined low probability zones.

Next, results for the use of crusher capacities are shown in figure 10, where it is possible to see that, once again, the stochastic schedule better uses the available capacities, preventing tonnages above the P50 simulated crushing capacities, as set in the model, with the P50 being the accepted

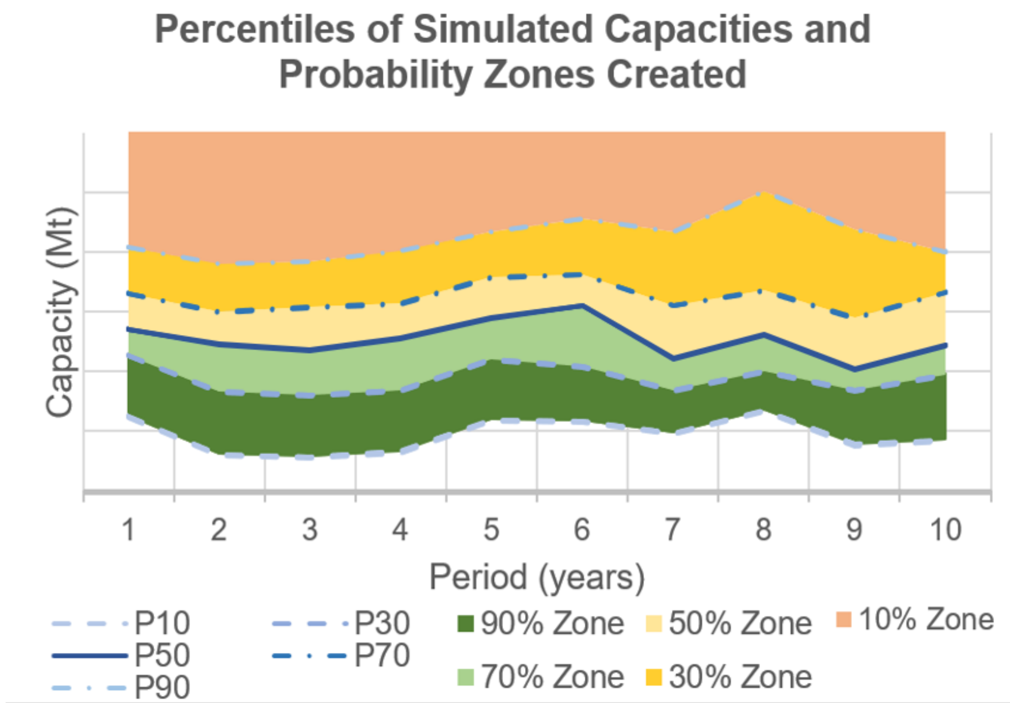


Figure 8: Visual representation of the probability zones defined by the percentiles of simulated mining capacities for Mine A.

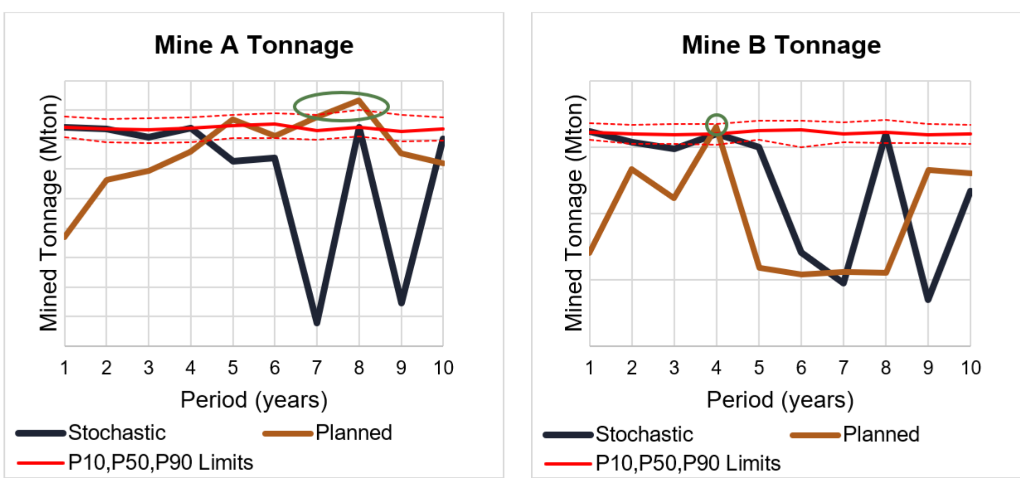


Figure 9: Risk profiles for the scheduled tonnage to be mined for the stochastic plan compared to the conventional schedule for both mines.

risk. Different from the conventional plan, the stochastic plan does not exceed the available targets and hence produces a realizable schedule, especially for crushers CH4 and CH5, where the previous plan forecasts tonnages that exceed the available capacities with around 10% more material for most of the periods for CH4 and has an excess of almost 80% of the total crusher capacity in period 4 for CH5.

For crushers CH1, CH2 and CH3, it is possible to see that the conventional plan did not forecast material above the available capacities. However, it is possible to notice a difference between the conventional plan and the P50-Plan forecasts, which come from the conventional schedule planning more material to be extracted than the available mining capacity, as shown in figure 9. Even though the planned material at these crushers does not exceed their capacity, the forecasts for years 5, 7 and 8 are still most likely not to realize for the conventional schedule since the mining capacities are exceeded.

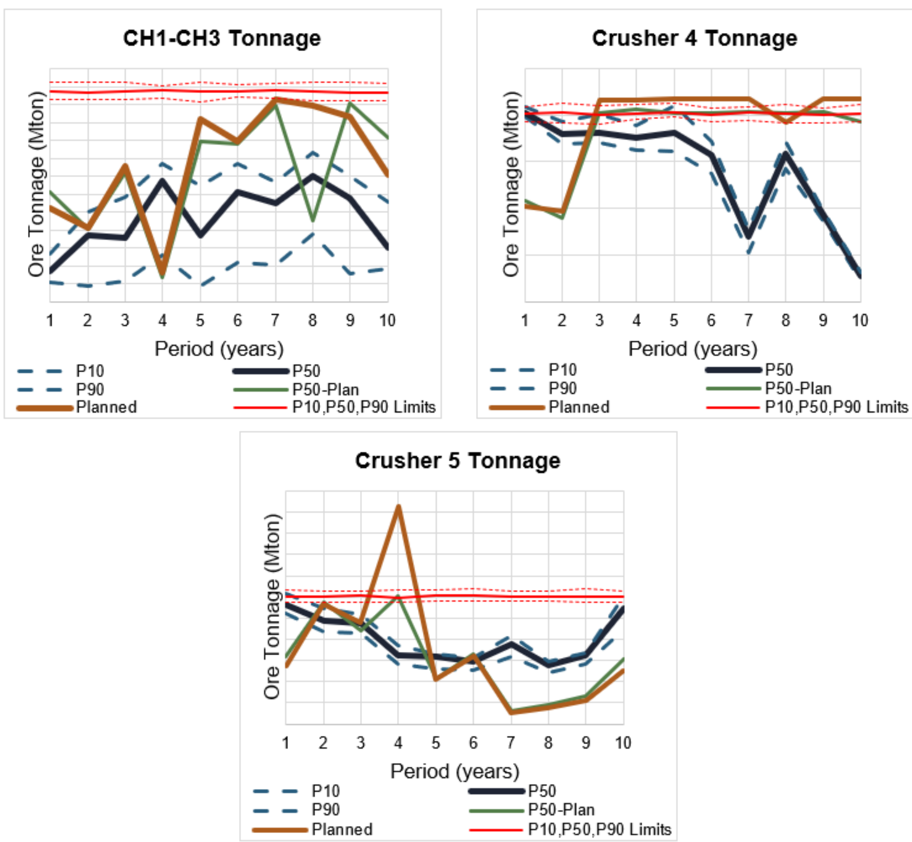


Figure 10: Risk profiles for the scheduled ore tonnages at the crushers CH1, CH2 and CH3 combined (upper left), crusher CH4 (upper right) and crusher CH5 (bottom) for the stochastic plan compared to the conventional schedule and P50-Plan from risk analysis. P10, P50 and P90 percentiles for the simulated capacities are shown in red.

Since the pit limits are the same, the total tonnages extracted are equal. However, the uncertainty in metal grades and material types affects the absolute tonnages of ore reaching the processing destinations. In different scenarios, a block can be classified as a different material type and have other destinations as possible processing destinations.

Moreover, the stochastic optimizer also uses dynamic cut-off grades as opposed to the fixed cut-offs used by the conventional plan, which also impacts the tonnages of metal recovered. Since the input to the conventional optimization is an estimated orebody model with smoothed grades, misrepresenting their distribution and variability, the current cut-offs used at the mining complex tend to be higher than the ones chosen by the stochastic optimizer, which also leads to more material sent to the waste and less metal recovered. figure 11 shows an example of the comparison between the dynamic cut-offs chosen by the stochastic optimizer for the high-grade sulphide material from Mine B and the fixed cut-off for the mills used in the conventional plan.

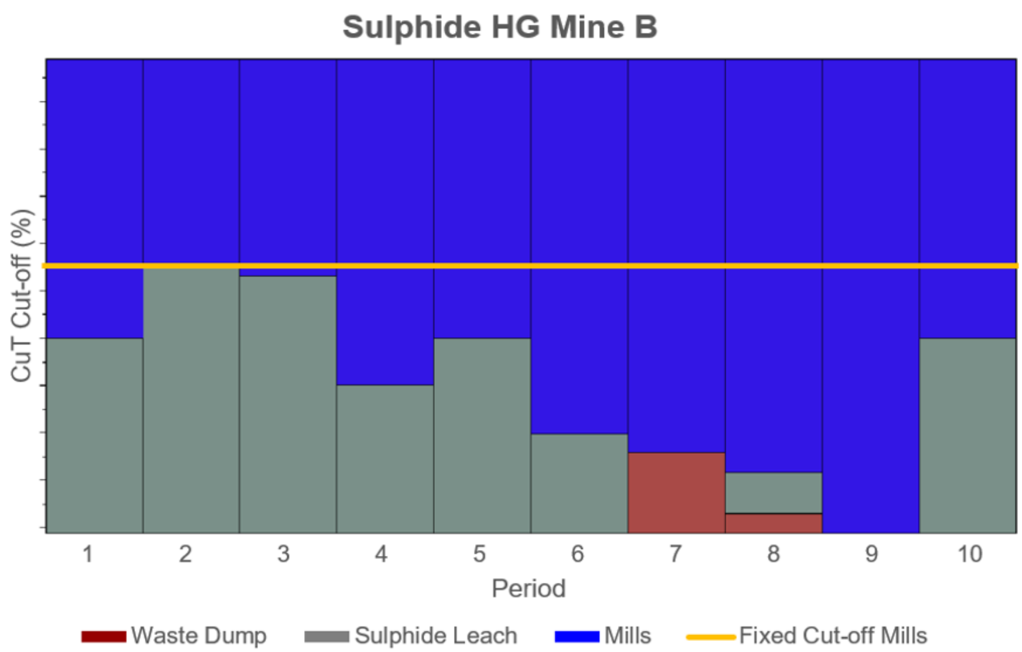


Figure 11: Comparison between the dynamic cut-off grades for the high-grade sulphide material from Mine B and the fixed cut-off used for the mills (yellow line).

As shown in figure 12, the stochastic plan manages to advance metal extraction and recovery, where 30% more metal is recovered at the mills, and 80% more metal is recovered at the leach pads in the first year of operations alone. While overall, 3% less metal is recovered at the mills, the leach pads show a total metal recovery 45% higher in the stochastic schedule. Those are direct results of the management of grade uncertainty and dynamic cut-offs since the stochastic plan sends over

105% more ore to the sulphide leach pad. Accounting for the uncertainty in equipment capacities ensures that the resulting stochastic schedule is more likely to be realized.

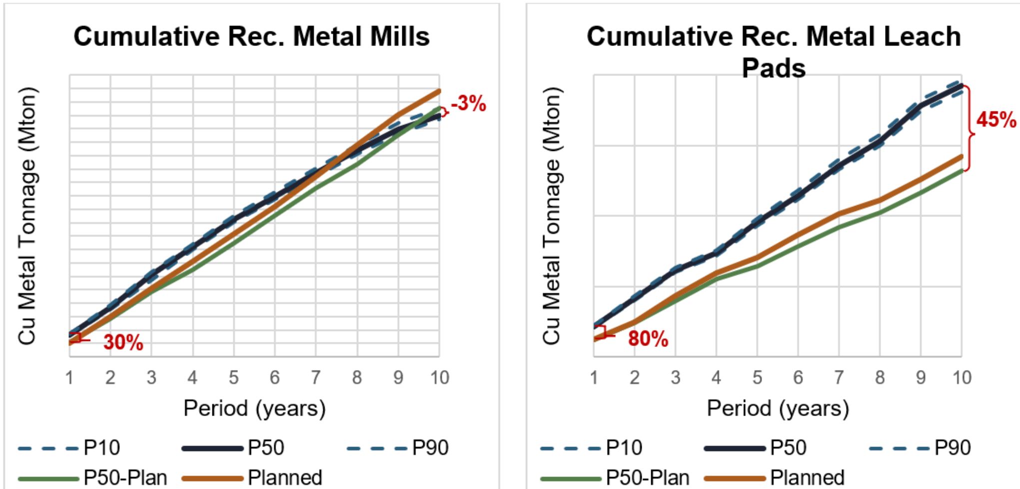


Figure 12: Risk profiles for the total recoverable copper metal at the three mills (left) and the leach pads (right).

All ore tonnages sent to the mills and leach pads in the stochastic and conventional schedules respect the total capacities at those destinations in all periods. The same can be said for the stockpiles, where the stockpiling limits are not exceeded in any period for the stochastic plan. Regarding waste tonnages, figure 13 shows that given the material uncertainty and dynamic cut-offs used in the stochastic optimizer, the stochastic schedule is found to send overall 19% less material to the waste dump, another reason for higher metal recovered and higher revenue.

The more substantial amount of recoverable gold in the stochastic solution with more material classified as ore and not being sent to the waste leads to a total of 45% higher cumulative discounted cash flow by the end of the 10 years considered in the study when compared to the P50-Plan, as shown in figure 14. The economic benefit is even more apparent within the first 3 years of operation when the stochastic solution generates a cumulative discounted cash flow 52% higher than the P50-Plan. As noted earlier, the stochastic approach that considers the supply and material uncertainty and the uncertainty in equipment capacities can blend different levels of uncertainty to reduce risk and increase value while producing schedules better suited to meet the available capacities and targets.

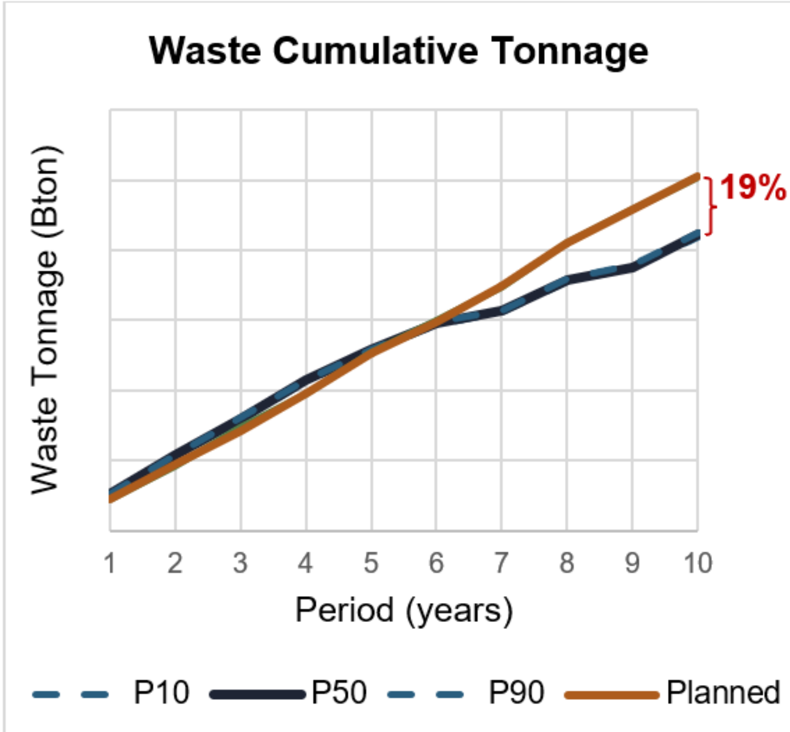


Figure 13: The cumulative tonnage of material extracted and sent to the waste dump in the stochastic and conventional schedules.

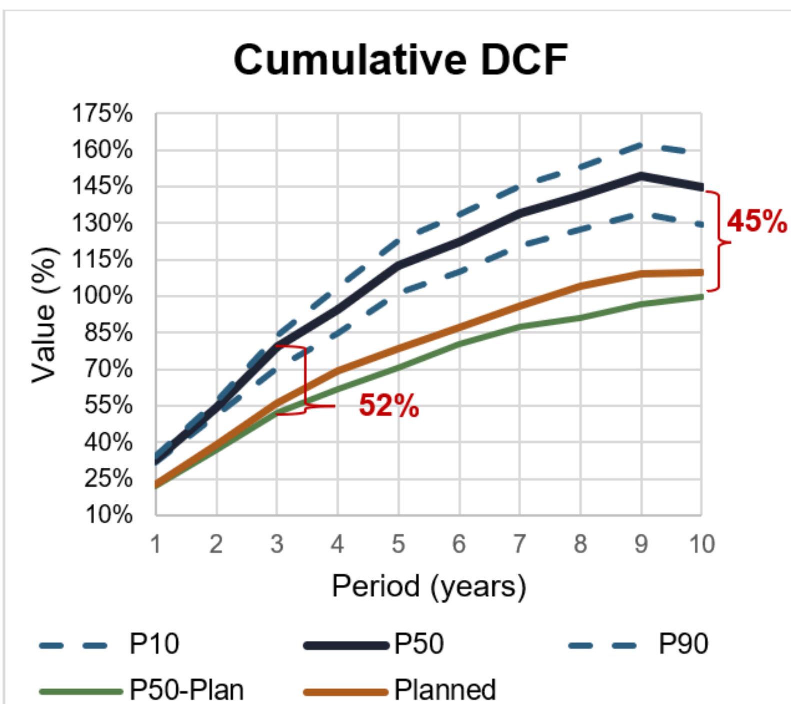


Figure 14: Cumulative discounted cash flow for the 10 years LOM).

4 Conclusion

This work applied the simultaneous stochastic optimization of mining complexes with supply and equipment capacity uncertainty to a large copper mining complex. The method was adapted to integrate the uncertainty related to different sources, such as equipment production. Risk quantification was done using stochastic simulations, which enable the assessment of the uncertainty and variability of data, especially for geological deposits, where limited information is obtained through drilling campaigns. Stochastic simulations are also used for modelling the uncertainty and variability of equipment production, where historical data was used to generate the realizations. The probability zones approach was used to include the equipment uncertainty in the framework and optimization process. Such an approach is characterized by giving freedom to the decision-maker to choose the level of risk accepted in the optimization process.

The current conventional schedule used at the mining complex presents significant shortfalls in the presence of uncertainty inherent to a mining operation with several complex interconnected components. However, the stochastic plan achieved shows to be able to manage the risk from the different sources, especially from the material supply, generating schedules better suited to respect the available capacities and targets while reducing risk and increasing the value of the project.

For the case study considered, results show close to 45% higher discounted cash flow by the end of the first 10 years, translating into tens (or even hundreds) of millions of dollars for large mining complexes. Moreover, more metal is produced, and less material is sent to the waste dump. At the same time, the same total tonnages are extracted, which are direct benefits from incorporating the uncertainty from different sources during the optimization process using the simultaneous stochastic optimization method.

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